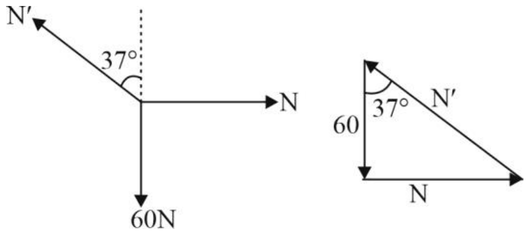
Solutions to JEE Main Full Length Test - 1 | JEE – 2024 | Gen-1 & 2

PHYSICS

SECTION – 1

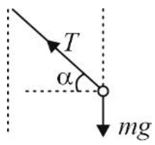
1.(D)



F.B.D. of system "sphere + block" $\frac{N}{60} = \frac{3}{4}$

$N = 45$ Newton

2.(A) $T\cos\theta = \frac{mv^2}{r}$



$T\sin\theta = mg$

$\tan\theta = \frac{gr}{v^2}$

$v^2 = \frac{gr}{\tan\theta} = \frac{gr}{\sqrt{3}}$; $\quad v = \left(\frac{gr}{\sqrt{3}}\right)^{1/2}$

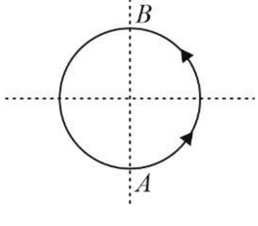
3.(B) $F_{\max} = M_{\text{system}} \times a_{\max}$

$= (1+2)\left(\frac{\mu(1)(g)}{1}\right) \quad = 3 \times 0.6 \times 10 = 18N$

4.(A) A-(II)
Velocity is increasing and positive.
B-(IV)
Velocity is negative and magnitude is decreasing.
C-(III)
Velocity is positive constant initially and then negative constant.
D-(I)
Velocity is a positive constant.

5.(B) $\omega_A = 0 \qquad \alpha = 2 \text{ rad/s}^2$

$\omega_B^2 - 0^2 = 2(\alpha)(\theta) \qquad = 2(2)(\pi)$



$\omega_B^2 = 4\pi \,\text{rad}^1/\text{sec}^2$

$a_r = \omega^2 x = 4\pi(1) = 4\pi \text{ m/s}^2$

$a_t = R\alpha = 1(2) = 2 \text{ m/s}^2$

$a_{\text{total}} = \sqrt{16\pi^2 + 4} = 2\sqrt{4\pi^2 + 1} \text{ m/s}^2$

6.(C) Centre of mass will lie on the line joining COM of both the rods.

7.(B) $\frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{v}{R}\right)^2 = mgh$

$h = \frac{3v^2}{4g} = \frac{3\times 7^2}{7\times 9.8} = 3.7\text{m}$

8.(A) Steady motion $\Rightarrow$ $a = 0$

$F_{net} = 0$; $F_v = F_B$

$6\pi\eta rv = \rho\cdot\frac{4}{3}\pi r^3 g$; $\eta = \frac{2}{9}\frac{\rho r^2 g}{v}$

9.(D) $\lambda_m T = \text{constant}$

10.(A) $\Delta T = 90°F = 50°C$

$\ell \propto \Delta T = \Delta\ell$

$\Rightarrow$ $\alpha = \frac{0.015}{10\times 50} = 3\times 10^{-5}\,°C^{-1}$

11.(C) Area covered with *V*-axis in expansion is area I + area II.

12.(A) $\Delta Q = \Delta U + \Delta W$

First process adiabatic expansion

$\Delta Q = 0,$ $\Delta W = +50J,$ $\Delta U = -50J$

Second process cooling at constant volume

$\Delta Q = -20J,$ $\Delta W = 0,$ $\Delta U = -20J$

$\Delta U_{total} = (-50) + (-20) = -70$

13.(D)

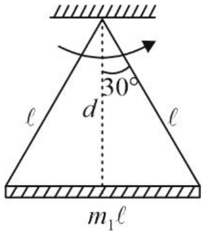
14.(D) $T = 2\pi\left(\frac{\ell}{g}\right)^{1/2}$

$g_{\text{at height}} < g_{\text{at surface}}$ $\therefore$ A is incorrect.

15.(B) The cylinder is at mean position now. When the block of mass M_0 is kept on it, it will have an amplitude of

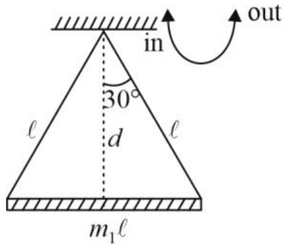
$M_0 g = Kx + \rho Axg$; $x = \frac{M_0 g}{K + A\rho g}$

16.(B) Physical pendulum



$T_1 = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{m\ell^2}{12} + m\left(\frac{\ell\sqrt{3}}{2}\right)^2}{mg\left(\frac{\ell\sqrt{3}}{2}\right)}}$

Simple pendulum



$$T_2 = 2\pi\sqrt{\frac{d}{g}} = 2\pi\sqrt{\frac{\frac{\ell\sqrt{3}}{2}}{g}}\ ; \qquad \left(\frac{T_1}{T_2}\right)^2 = \frac{10}{9}$$

17.(C) We know that $\quad P_B = P_A + \frac{1}{2}\rho\omega^2 a^2$

$$P_D = P_A + \rho g a$$

$$P_C = P_D + \frac{1}{2}\rho\omega^2 a^2 = P_A + \rho g a + \frac{1}{2}\rho\omega^2 a^2$$

Therefore, $\quad P_C > P_A$ for all values of ω and $P_B > P_D$ only if $\omega > \sqrt{\frac{2g}{a}}$

18.(A) Volume will remain conserved

$$\pi(4R^2)(9R) = \frac{4}{3}\pi R'^3$$

$$R'^3 = 27R^3; \qquad R' = 3R$$

$$I_1 = \frac{M(2R)^2}{2}; \qquad I_2 = \frac{2}{5}M(3R)^2$$

$$\frac{I_1}{I_2} = \frac{4}{2}\ \frac{5}{18} = \frac{5}{9}$$

19.(C) Velocity with which the liquid leaves the hole, $v = \sqrt{2gh}$

Time taken by any particle of the liquid to travel from the hole to the ground, $t = \sqrt{\frac{2(H-h)}{g}}$

Therefore, $\quad L = vt = \left(\sqrt{2gh}\right)\left(\sqrt{\frac{2(H-h)}{g}}\right) = 2\sqrt{h(H-h)}$

For $h = \frac{2H}{5}$, $\quad L = 2\sqrt{\left(\frac{2H}{5}\right)\left(\frac{3H}{5}\right)} = \frac{2\sqrt{6}}{5}H$

20.(A) $V_{\text{orbital}} = \left(\frac{GM_E}{r}\right)^{1/2}$

SECTION – 2

1.(2) $(V-u) = 10$

$$(V+u) = 14$$

$$2u = 4$$

$$u = 2 \text{ kmph}$$

2.(200) Distance (D) $= 2R + 2\pi R \times \frac{2}{3}$

Time $= \frac{D}{v}$

3.(3) Time of flight $= \frac{2u\sin\theta}{g} = 2 \times \frac{20}{10} \times \frac{\sqrt{3}}{2} \qquad = 2\sqrt{3}\,\text{sec}$

Required time $= \frac{T}{2} = \sqrt{3}\,\text{sec}$

4.(1) $2v = 100 \times 0.02$; $\qquad v = 1$ m/s

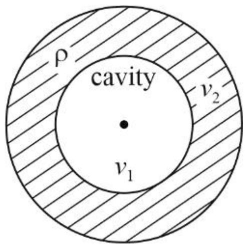
5.(1) $\rho v_2 = 10$ (in air)

$\rho v_2 - \rho_\omega (v_1 + v_2) = 7.5$ (in water)

$\rho_\omega (v_1 + v_2) = 2.5$

$\frac{\rho_\omega (v_1 + v_2)}{\rho (v_2)} = \frac{2.5}{10} = \frac{1}{4}$

$\frac{1000}{8000}\left(\frac{v_1}{v_2} + 1\right) = \frac{1}{4}; \qquad \frac{v_1}{v_2} = 1$



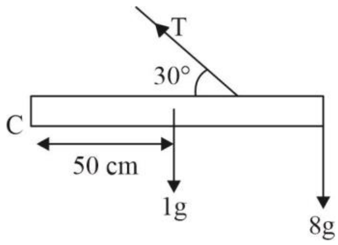
6.(22) After a long time, the rate of heat loss from the window becomes equal to the rate of heat supply by the heater, and the hence the temperature inside the room becomes constant

$$\frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{L} \quad \Rightarrow \quad 1920 = \frac{(1)(1.2)(T_1 - 14)}{0.5 \times 10^{-2}} \quad \Rightarrow \quad T_1 = 22\ ^\circ\text{C}$$

7.(300) $\tau_C = 0$

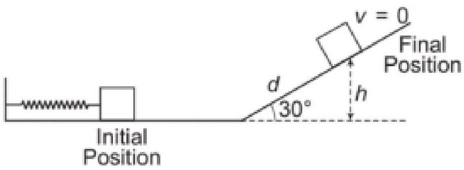
$(T \sin 30^\circ) \times 60 - 2g \times 50 + 8g \times 100$

$T = \frac{9000}{30} = 300\,N$



8.(2) In the final position, block will stop for a moment and then it will return back.

In the initial position system has only spring potential energy $\frac{1}{2}kx^2$ and in the final position it has only gravitional potental energy



Since, all surfaces are smooth, therefore mechanical energy will remain conserved.

$\Rightarrow \qquad E_i = E_f$ or $\frac{1}{2}kx^2 = mgh = mg\left(\frac{d}{2}\right)$

where $\quad h = d\sin 30^\circ = \frac{d}{2} \qquad \Rightarrow \qquad d = \frac{kx^2}{mg}$

Substituting the values we have,

$$d = \frac{(10)(2)^2}{(2)(10)} = 2m$$

9.(10) $V_{esc} = \left(\frac{2GM}{R}\right)^{1/2}$

$$(V_{esc})_P = \left(\frac{2GM_P}{R_P}\right)^{1/2} \qquad (V_{esc})_E = \left(\frac{2GM_E}{R_E}\right)^{1/2}$$

$$V_{esc\ p} = \left(\frac{2G(10ME)}{R_E/10}\right)^{1/2} = 10 \times (V_{esc})_E$$

10.(5) $\Delta L = \frac{\left(\frac{mg}{2}\right)L}{AY} = \frac{\left(\frac{10}{2}\right)(0.4)}{10\times10^{-4}\times4\times10^{11}}$

$$= 5\times\frac{4}{100}\times\frac{10^4}{4}\times10^{-11} = 5\times10^{-9}m$$

CHEMISTRY

SECTION – 1

1.(A) Lactic acid is weak acid.
Mixture of calcium lactate and lactic acid acts as acidic buffer solution having [conjugate base] = [weak acid].
Hence $pH = pK_a$

pH of calcium lactate is calculated by $pH = 7 + \frac{1}{2}pK_a + \frac{1}{2}\log C = 7 + \frac{1}{2}(5) + \frac{1}{2}\log(0.01)$

2.(A) According to kinetic theory of gases, gases expand and occupy all the space available to them because of no force of attraction between gas particles at ordinary temperature and pressure.

3.(B)

	Titration			Indicator
(P)	$H_2C_2O_4$ vs Acidified $KMnO_4$	:	**(III)**	$KMnO_4$ (self-indicator)
(Q)	$FeSO_4$ vs Acidified $K_2Cr_2O_7$	:	**(IV)**	Diphenyl amine
(R)	$CuSO_4$ vs KI	:	**(I)**	Starch
(S)	$H_2C_2O_4$ vs NaOH	:	**(II)**	Phenolphthalein

4.(D) Fluorine does not exhibit positive oxidation state i.e., fluorine shows only one non-zero oxidation state hence can't involve in disproportionation.

(a)	Reduction of F and oxidation of O	**(b)**	Comproportionation
(c)	Reduction of F and oxidation of O	**(d)**	Disproportionation

5.(B) Orthoboric acid is not protic acid, it is Lewis acid. All other are protic acid.

6.(C) Synthesis of ammonia is exothermic hence on increasing temperature rate of reaction increases but overall equilibrium shift in backward direction hence yield of ammonia decreases.

7.(A) In upper atmosphere O_2 is converted to O_3 photochemically hence there is dynamic equilibrium between production and decomposition of ozone in the stratosphere.

$3O_2(g) \longrightarrow 2O_3(g) \qquad \Delta_r H > 0$ and $\Delta_r S < 0$

Hence conversion of O_2 to O_3 is thermodynamically non-spontaneous.

8.(C) $CH_4(g) + 2O_2(g) \longrightarrow CO_2(g) + 2H_2O(\ell)$

$\Delta n_g = 1 - 3 = -2$

$\Delta H^\circ = \Delta U^\circ + \Delta n_g RT$: $\qquad \Delta H^\circ < \Delta U^\circ$ because $\Delta n_g < 1$.

9.(B) Ion-dipole forces are not Vander Waals forces, hydrogen bond is a powerful force in determining the structure and properties of protein.

10.(B) Molecular shape of XeF_4 is square planar but its hybridization state is sp^3d^2.

11.(C) Order of first ionization enthalpy is Na < Mg < Al < Si.

12.(C) $4d_{z^2}$: both spherical nodes and conical nodes.

$3d_{x^2-y^2}$ or $3d_{xy}$: only planar nodes.

$3d_{z^2}$: only conical nodes.

$2p_z$: only planar nodes.

13.(D) Ethene is thermodynamically less stable than ethane. Strength of multiple bond is greater than single bond.

14.(D) Only conformational isomers can't be separated due to inter conversion of conformers because of almost free rotation about carbon-carbon single bond.

15.(D) Formation of only one o-disubstituted derivative can be explained by using concept of oscillating nature of double bonds in benzene i.e., two identical contributing structures.

16.(B) (P) is cis-2-Butene while (Q) is trans-2-Butene.

17.(D) Latest technique for isolation, purification and separation of organic compounds is chromatography.

18.(C) Acetic acid is used to neutralize alkaline nature of sodium extract.

19.(C) Refer NCERT chapter Some Basic Concepts and Techniques in Organic Chemistry.

20.(B) $B_2 \Rightarrow \sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2px}^1\pi_{2py}^1$; Bond order is 1 and single bond is π-bond.

$C_2 \Rightarrow \sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2px}^2\pi_{2py}^2$; Bond order is 2 and double bond is made up of two pi- bonds.

$N_2 \Rightarrow \sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2px}^2\pi_{2py}^2, \pi_{2pz}^2$; Bond order is 3 and it is equal to one sigma and two pi-bonds.

$O_2 \Rightarrow \sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \sigma_{2px}^2, \pi_{2px}^2\pi_{2py}^2, \pi_{2px}^{*1}\pi_{2py}^{*1}$; Bond order is 2 and it is equal to one sigma and one pi-bond.

s-p mixing is observed in case of B_2, C_2 and N_2.

SECTION – 2

1.(54) $Me_2NH + H_2O \rightleftharpoons Me_2NH_2^+ + HO^-$

0.02	Ex	0	0.1
0.02 (1 – x)		0.02x	0.1 + 0.2x

$$5.4\times10^{-4} = \frac{0.02x+0.1}{0.02} \Rightarrow x = \frac{5.4\times10^{-4}}{0.1} = 5.4\times10^{-3}$$

2.(307) According to I Law of thermodynamics.

$$\Delta U = q + w = 701 + (-394) = 307\,J$$

3.(600) Reaction of (A) with ammonical $AgNO_3$ indicate that it is an alkyne.

$$14n - 2 = 54 \Rightarrow n = \frac{56}{14} = 4$$

(A) is $CH_3-CH_2-C\equiv CH$

Molecular mass of $B = 374\,g/mole$

$x = 592.59$

Molecular mass of $C = 58\,g/mole$

$y = 7.40$

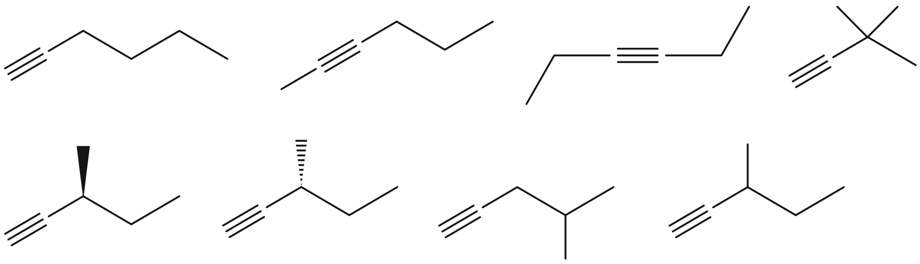
$x + y = 599.99 = 600$

4.(1563)

Charge of electron $= 1.6\times10^{-19}C$

Number of electrons $= \dfrac{2.5\times10^{-16}}{1.6\times10^{-19}} = 1562.5 \simeq 1563$

5.(8)



6.(100) Mole of Ca^{2+} = Mole of $SO_4^{2-} + \frac{1}{2}$(mole of HCO_3^-) $= \left(\frac{96}{96}\right) + \left(\frac{1}{2} \times \frac{183}{61}\right) = 2.5$

Mass of $Ca^{2+} = (2.5 \times 40)$ per 10^{-6} gm solution

Conc. of $Ca^{2+} = 100\,ppm$

7.(18) $(N \equiv C)_2C = C(C \equiv N)_2$

Ethene-1, 1, 2, 2-tetracarbonitrile

Number of σ-bonds = 9

Number of π-bonds = 9

8.(18) Total number of electrons in a subshell $= 2(2\ell + 1)$; $\ell = 4$ for g-subshell.

9.(3) Compounds having C, N and halogen can gives positive Lassaigne's test for both nitrogen and halogen.

$CH_3NH_2 \cdot HCl$, NBS and $NH_2CONHNH_2 \cdot HBr$ gives positive Lassaigne's test for both nitrogen and halogen.

10.(11) Compound (P) contains two terminal double bonds because of formation of two mole of CH_2O.

Compound (P) is $CH_2 = C(CH_3) - CH = CH_2$.

Total 11 atoms of (P) are lying in the same plane.

MATHEMATICS

SECTION-1

1.(C) $\dfrac{2xy}{x+y} = 6^{20}$

$xy = (x+y)2^{19}3^{20}$

$\left(x-2^{19}3^{20}\right)\left(y-2^{19}3^{20}\right) = 3^{40}\cdot 2^{38} \quad \Rightarrow \alpha = 40, \ \beta = 38$

$(40+1)(38+1) = 1599$

$\dfrac{1599-1}{2} = 799$

2.(D) Here $z_1 + z_2 + z_3 = 2(2-i)$ then $z_2 + z_3 = 3-i$

Again $z_1 z_2 z_3 = 1-3i$ then $z_2 z_3 = 2-i$ then $z_2 = 1$

and $z_3 = 2-i$.

3.(B) $E = \left(\sqrt{9^{|x-2|}} + \left[4.3^{|x-2|} - 9\right]^{1/5}\right)^7$

$= \left(3^{|x-2|} + \left(4.3^{|x-2|} - 9\right)^{1/5}\right)^7$

$\equiv (x+y)^7$

$T_6 = {}^7C_5 x^{7-5} y^5 = 567$

$567 = 21\left(3^{2|x-2|}\right)\left\{\left((4)3^{|x-2|} - 9\right)\right\}$

Let $t = 3^{|x-2|}$

$\therefore \ 4t^3 - 9t^2 - 27 = 0$

$\therefore \ t = 3$

$3^{|x-2|} = 3$

$\Rightarrow |x-2| = 1$

$\therefore \ x = 3, 1$

4.(A) Required sum $\dfrac{(1+x)^{1000}\left(\left(\dfrac{x}{1+x}\right)^{1001} - 1\right)}{\left(\dfrac{x}{1+x} - 1\right)} = (1+x)^{1001} - x^{1001}$

Coefficient of x^{25} is

$\therefore \ {}^{1001}C_{25}$

5.(D) Let $\left(5-2\sqrt{6}\right)^n = f' \qquad 0 < f' < 1$

$I + f + f'$ = even integer

$\therefore \qquad f + f' = 1$ or $f' = 1 - f$

$I + f = \left(5+2\sqrt{6}\right)^n$

$I = \left(5+2\sqrt{6}\right)^n - f = \dfrac{1}{\left(5-2\sqrt{6}\right)^n} - f$

$I = \dfrac{1}{1-f} - f$

6.(C) If $L: ax+by=c$

Reflection through *y*-axis

$M: -ax+by=c \qquad m_M = \dfrac{a}{b}$

Reflection through *x*-axis

$N: ax-by=c \qquad m_N = \dfrac{a}{b}$

7.(B)

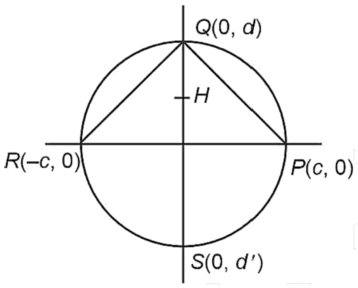


Image of orthocentre in any sides of a triangle lies on circumcircle of the triangle.
H is the image of d' in *x*-axis.

8.(C) $L: (x+y+1)+b(2x-3y-8)=0$ are concurrent at $(1,-2)$. Line through $A(1,-2)$ which is farthest from $B(2,2)$ is perpendicular to *AB*.

Hence, area is $\dfrac{49}{8}$ sq. unit.

9.(C) Let circle be $x^2+y^2+2gx+2fy+c=0$ passes through (1, 2)

$\Rightarrow \quad 2g+4f+c+5=0$

From orthogonal condition $2(0+0)=c-16$

$\Rightarrow \quad c=16$

10.(D) Equation of the tangent at (1, 3) to C_1 is

$x+3y=10 \qquad \ldots$(i)

Suppose $T(x_1, y_1)$ is the point where the tangents to C_2 at points *A* and *B* meet.

Hence, equation of the chord *AB* is $xx_1+yy_1-16=0$

$\dfrac{x_1}{1}=\dfrac{y_1}{3}=\dfrac{-16}{-10}=\dfrac{8}{5} \Rightarrow x_1=\dfrac{8}{5},\ y_1=\dfrac{24}{5}$

11.(B) Common chords will pass through the centre of the respective circles.

$\Rightarrow \quad 2gx+2fy+c+9=0$ pass through (0, 0)

$\Rightarrow \quad c=-9$

Also, $2gx+2fy+c+4x-2y-4=0$ pass through (2, –1)

$\Rightarrow \quad 4g-2f-3=0$

Also, $2gx+2fy+c-2x+6y-6=0$ pass through (–1, 3)

$\Rightarrow -2g+6f+5=0$

$\therefore \quad g=\frac{2}{5}, f=\frac{-7}{10}$

12.(D) Clearly, points P, Q, R are on the line $y=x+1$, have their chord of contacts should be concurrent.

$\therefore$ Area $=0$

13.(A) Equation of chord of contact is $hx+3ky=12$.

Let tangents are drawn from $T(h,k)$ to the ellipse $x^2+3y^2=12$.

Equation of chord of contact is $hx+3ky=12$.

which is tangent on $x^2+9y^2=9$

$\Rightarrow 9\left(\frac{h^2}{9k^2}\right)+1=\frac{16}{k^2}$

$\Rightarrow h^2+k^2=16$ which is the director circle of $x^2+3y^2=12$

14.(C) Line joining point of intersection of tangent and mid-point of points of contact is parallel to axis of parabola.

15.(A) Using $PS+PS'=2a$ reflection property total distance $=4(2a)=8a$.

16.(A) Tangent at t of $xy=c^2$

$\Rightarrow \quad y=-\frac{x}{t^2}+\frac{2c}{t}$

And normal to $y^2=8x$

$\Rightarrow \quad y=mx-2am-am^3$

$\Rightarrow \quad m=\frac{-1}{t^2}$ and $\frac{2c}{t}=-2am-am^3$

$\Rightarrow \quad 2ct^5-2at^4-a=0 \quad (c=3, a=2)$

$\Rightarrow \quad 6t^5-4t^4-2=0$

$t=1$

17.(B) Rearranging as $(x-3)^2+(y-1)^2=\left(\frac{15x-8y+13}{17}\right)^2$

$\therefore \quad$ Focus is (3, 1)

Directrix is $15x-8y+13=0$

18.(B) New curve will be $\frac{x^2}{9}+\frac{y^2}{25}=1$

19.(B) $\frac{\alpha^3+1}{\alpha^5-\alpha^4-\alpha^3+\alpha^2}=\frac{(\alpha+1)(\alpha^2-\alpha+1)}{(\alpha+1)(\alpha^2-\alpha)^2}=\frac{4}{9}$

20.(C) $\dfrac{9\alpha_9-\alpha_{10}}{6\alpha_8}=\dfrac{9\left(a^9-b^9\right)-\left(a^{10}-b^{10}\right)}{6\left(a^8-b^8\right)}$

$$=\frac{a^8\left(9a-a^2\right)-b^8\left(9b-b^2\right)}{6\left(a^8-b^8\right)}$$

$$=\frac{18\left(a^8-b^8\right)}{6\left(a^8-b^8\right)}=3$$

SECTION – 2

1.(14) $|z_1|+|z_2|+|z_3|\le 14$

2.(12) Let $w=\cos 12°+i\sin 12°+\cos 48°+i\sin 48°$

$$=2\cos 18°\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)=2\cos 18°\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)$$

$$z=w^2=\cos^2 18°\left(2+2\sqrt{3}i\right)$$

$$\Rightarrow\ \text{Im}(z)=2\sqrt{3}\cos^2 18°$$

3.(1) $x^2-2pxy-y^2=0$

Bisector

$$\Rightarrow\ \frac{x^2-y^2}{2}=\frac{xy}{-p}$$

i.e., $px^2+2xy-py^2=0\equiv x^2-2qxy-y^2=0$

$\therefore\ pq=-1$

4.(4) Angle between DCTs is given by

$$2\sin^{-1}\left(\frac{|r_1-r_2|}{d}\right)=\left|2\sin^{-1}\left(\frac{1}{2}\right)\right|=\frac{\pi}{3}$$

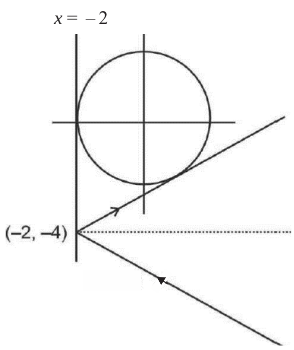
5.(12) For tangent from (–2, –4)

$$-4=m(-2)+2\sqrt{1+m^2}$$

$$\Rightarrow\ (m-2)^2=1+m^2$$

$$\Rightarrow\ -4m+3=0$$

$$m=\frac{3}{4}$$



So, slope of incident ray $=-\dfrac{3}{4}$

6.(8) Length of chord of contact $= \frac{1}{a}\sqrt{y_1^2 - 4ax_1}\ \sqrt{y_1^2 + 4a^2} = \frac{1}{1}\sqrt{(0)^2 - 4(-4)} \cdot \sqrt{(0)^2 + 4(1)^2} = 8$

7.(6) Use the property $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{3} - \frac{1}{6} = -\frac{1}{6}$

8.(5) Equation of hyperbola $(2x - y - 3)(3x + y - 7) = k$

Put (1,1) we get $k = 6$

$\therefore$ Equation of hyperbola is $(2x - y - 3)(3x + y - 7) = 6$

$$6x^2 - 3xy - 9x + 2xy - y^2 - 3y - 14x + 7y + 21 = 6$$

$$6x^2 - xy - y^2 - 23x + 4y + 15 = 0$$

$$a = -1,\ b = -23,\ c = 4,\ d = 15$$

9.(2) $\because\ \frac{x^2 - 3x + \lambda}{x - 2} = y$

$\therefore\ x^2 - (3 + y)x + \lambda + 2y = 0$

For all $x, D \geq 0$

$$(3 + y)^2 - 4(\lambda + 2y) \geq 0$$

$$y^2 - 2y + 9 - 4\lambda \geq 0$$

$\therefore\ D \leq 0$

$\Rightarrow\ \lambda \leq 2$

But $\lambda = 2$ is not acceptable.

$\therefore\ k \in (-\infty, 2)$

10.(15) $\because\ \cot\frac{\theta}{2} = \frac{1 + \cos\theta}{\sin\theta}$

$$\therefore\ \cot\left(\frac{15}{2}\right)^\circ = \frac{1 + \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$= \sqrt{6} + \sqrt{3} + \sqrt{2} + \sqrt{4}$$

$\therefore\ n_1 + n_2 + n_3 + n_4 = 15$